SAFE HANDS & IIT-ian's PACE

LEAP TEST# 09 (JEE) ANS KEY Dt. 23-12-2023

PHYSICS		CHEMISTRY		MATHS	
Q. NO.	[ANS]	Q. NO.	[ANS]	Q. NO.	[ANS]
1	B	31	C	61	B
2	B	32	A	62	C
3	A	33	B	63	B
4	B	34	A	64	A
5	C	35	D	65	C
6	B	36	A	66	B
7	C	37	C	67	A
8	A	38	C	68	D
9	A	39	C	69	A
10	A	40	D	70	A
11	C	41	B	71	C
12	B	42	D	72	A
13	A	43	D	73	C
14	C	44	B	74	D
15	B	45	D	75	C
16	A	46	B	76	A
17	B	47	B	77	A
18	B	48	A	78	C
19	C	49	C	79	A
20	D	50	C	80	C
21	2.55	51	48	81	12
22	1	52	1	82	1
23	0.07	53	347	83	6
24	0.8	54	10	84	3
25	280	55	2	85	0

See Physics & Maths solutions on next page.....

SAFE HANDS & PACE
LT 09 (JEE) PHYSICS SOLUTIONS

: ANSWER KEY :

1)	b	2)	b	3)	a	4)	b	21)	2.55
5)	c	6)	b	7)	c	8)	a	22)	1
9)	a	10)	a	11)	c	12)	b	23)	0.07
13)	a	14)	c	15)	b	16)	a	24)	0.8
17)	b	18)	b	19)	c	20)	d	25)	280

: HINTS AND SOLUTIONS :

Single Correct Answer Type

1 **(b)**

2 **(b)**

3 **(a)**

Rate of heat loss per unit area due to radiation

$i.e.$ emissive power $e = \varepsilon\sigma(T^4 - T_0^4)$

$= 0.6 \times \frac{17}{3} \times 10^{-8} \times [(400)^4 - (300)^4]$

$= 3.4 \times 10^{-8} \times (175 \times 10^8) = 3.4 \times 175$

$= 595\, J/m^2 \times s$

4 **(b)**

$Q = mL = KA\frac{(\theta_1 - \theta_2)}{l}t \Rightarrow m$

$= \frac{1}{L} \times KA\frac{(\theta_1 - \theta_2)}{l} \times t$

$= \frac{1}{80} \times 0.2 \times 4 \times \frac{(100-0)}{\sqrt{4}} \times 10$

$\times 60 \left[\because l^2 = 4 \Rightarrow l = \sqrt{4}\right]$

$= \frac{0.2 \times 4 \times 100 \times 600}{80 \times 2} = 300\, gm$

5 **(c)**

Heat required to convert 10 g of ice at 0°C to water at 0°C

$Q_1 mL = 10 \times 80$ cal

Heat required to raise the temperature of water from 0°C to 20°C

$Q_1 = cm\theta = 1 \times 10 \times 20 = 200$ cal

Total heat required

$= Q_1 + Q_2 = 800 + 200 = 1000$ cal

6 **(b)**

7 **(c)**

The volume of the metal at 30°C is

$V_{30} = \frac{\text{loss of weight}}{\text{Specific gravity } \times g} = \frac{(45-25)g}{1.5 \times g}$

$= 13.33\, cm^3$

Similarly, Volume of metal at 40°C is

$V_{40} = \frac{(45-27)g}{1.25 \times g} = 14.40 cm^3$

Now, $V_{40} = V_{30}[1 + \gamma(t_2 - t_1)]$

$\Rightarrow \gamma = \frac{V_{40} - V_{30}}{V_{30}(t_2 - t_1)} = \frac{14.40 - 13.33}{13.33(40-30)}$

$= 8.03 \times 10^{-3}/°\text{C}$

$\therefore$ Coefficient of linear expansion of the metal is

$\alpha = \frac{\gamma}{3} = \frac{8.03 \times 10^{-3}}{3} = 2.6 \times 10^{-3}/°\text{C}$

8 **(a)**

The Stefan's law,

$E = \sigma T^4$ where σ is Stefan's constant,

Given, $T_1 = 27°\text{C} = 27 + 273 = 300$ K

$T_2 = 84°\text{C} = 273 + 84 = 357$ K

$\therefore \quad \frac{E_1}{E_2} = \frac{T_1^4}{T_2^4}$

$= \frac{(300)^4}{(357)^4} = \frac{1}{(1.19)^4}$

Rate of increase of energy is

$\frac{E_2}{E_1} = (1.19)^4 = 2$

9 **(a)**

10 **(a)**

Change in volume, $\Delta V = V\gamma\,\Delta t$

$\Rightarrow \quad 0.24 = 100 \times \gamma \times 40$

$\gamma = \frac{0.24}{100 \times 40}$

$= 0.00006 = 6 \times 10^{-5}$

$\alpha = \frac{\gamma}{3}$

$\Rightarrow \quad \alpha = 2 \times 10^{-5}°\text{C}^{-1}$

11 **(c)**

12 **(b)**

Substances are classified into two categories

(i) water like substances which expand on solidification.

(ii) CO_2 like (Wax, Ghee $etc.$) substances which contract on solidification.

Their behaviour regarding solidification is opposite.

Melting point of ice decreases with rise of pressure but that of wax etc increases with increase in pressure. Similarly ice starts forming from top to downwards whereas wax starts its formation from bottom to upwards

13 **(a)**

14 **(c)**

Infinite thermal capacity implies that there would be practically no change in temperature whether heat is taken in or given out.

15 **(b)**

Rate of heat flow $\left(\frac{Q}{t}\right) = \frac{k\pi r^2(\theta_1-\theta_2)}{L} \propto \frac{r^2}{L}$

$\therefore \frac{Q_1}{Q_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{l_2}{l_1}\right) = \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{1}\right) = \frac{1}{2} \Rightarrow Q_2 = 2Q_1$

16 **(a)**

According to Stefan's law

$E \propto T^4$

$\frac{E'}{E} = \left(\frac{3T}{T}\right)^4$ or $E' = 81E$

17 **(b)**

Here, $\gamma_{ag} = 10.30 \times 10^{-4}{}^\circ C^{-1}$

$\gamma_{am} = 10.06 \times 10^{-4}\ {}^\circ C^{-1}$

$\alpha_a = 9 \times 10^{-6}{}^\circ C^{-1}, \alpha_m = ?$

Now, $\gamma_r = \gamma_{ag} + g_{glass} = \gamma_{am} + g_m$

$\therefore 10.30 \times 10^{-4} + 3 \times 9 \times 10^{-6}$

$= 10.06 \times 10^{-4} + g_m \quad [\because g_g = 3 \times \alpha_a]$

$\therefore g_m = (10.30 + 0.27 - 10.06)10^{-4}$

$= 0.51 \times 10^{-4}$

$\alpha_m = \frac{1}{3}g_m = \frac{0.50 \times 10^{-4}}{3}$

$= 0.17 \times 10^{-4} = 17 \times 10^{-6}{}^\circ C^{-1}$

18 **(b)**

19 **(c)**

How can you forget that ln(σ) will be negative

20 **(d)**

According to Wien's displacement law,

$\lambda_m T = b \text{ or } \lambda_m \propto \frac{1}{T}$

Where b is Wien's constant whose value is 29×10^{-3} mK

$\frac{(\lambda_m)_S}{(\lambda_m)_F} = \frac{T_F}{T_S}$

Or $T_F = T_S \times \frac{(\lambda_m)_S}{(\lambda_m)_F} = 5500\text{ K} \times \frac{(5.5\times10^{-7}\ m)}{(11\times10^{-7}\ m)}$

$= 2750$ K

Integer Answer Type

21 **(2.55)**

Heat gained by water and container = Heat supplied by heater

$\therefore (m_w \times c_w \times \Delta\theta_1) + (m_c \times c_c \times \Delta\theta_1) = P \times t_1$

$\therefore \left(\frac{1}{2} \times 4200 \times 3\right) + (m_c \times c_c \times 3)$

$= 10 \times 15 \times 60$

$\therefore 6300 + 3(m_c \times c_c) = 9000$

$\therefore (m_c \times c_c) = \frac{2700}{3} = 900$

In case of oil,

$(m_0 \times c_0 \times \Delta\theta_2) + (m_c \times c_c \times \Delta\theta_2) = P \times t_2$

$\therefore (2 \times c_0 \times 2) + (m_c \times c_c \times 2) = (10 \times 20 \times 60)$

$\therefore 4c_0 + (900 \times 2) = 12000$

$\therefore c_0 = 2.55 \times 10^3 \text{J kg}^{-1}\text{ K}^{-1}$

22 **(1)**

Because the sliding plug stays in the connecting pipe the pressure in both the vellels at the level of the pipe must be the same

$h_{20}d_{20} = h_{80}d_{80} \Rightarrow \frac{h_{20}}{h_{80}} = \frac{d_{80}}{d_{20}}$

$\Rightarrow \frac{d_0(1-80\gamma)}{d_0(1-20\gamma)} = 0.94$

23 **(0.07)**

$\Delta d = \alpha(t_2 - t_1) \times d_1$

$= 12 \times 10^{-6} \times (80 - 20) \times 100$

$= 720 \times 10^{-6} \times 100$

$\therefore \Delta d = 0.072$ cm

≈ 0.07 cm

.....(Rounding off to 2 decimal places)

24 **(0.8)**

$Q = mc\theta$

$\therefore \frac{dQ}{dt} = mc\frac{d\theta}{dt}$

Also, $m = \rho V$

$\therefore \frac{dQ}{dt} = \rho\, Vc\frac{d\theta}{dt}$

As, equal volumes are cooled under identical conditions,

$\left(\frac{dQ}{dt}\right)_1 = \left(\frac{dQ}{dt}\right)_2$ and $V_1 = V_2 = V$

$\therefore \frac{\rho_2 Vc_2 \frac{dQ}{dt}_2}{\rho_1 Vc_2 \frac{dQ}{dt}_1} = 1$

$\therefore \frac{\rho_2}{\rho_1} \times \frac{c_2}{c_1} \times \frac{dt_1}{dt_2} = 1$

$\therefore \frac{\rho_1}{\rho_2} = \frac{c_2}{c_1} \times \frac{dt_1}{dt_2} = \frac{4}{3} \times \frac{510}{850} = \frac{4 \times 3}{3 \times 5} = 0.8$

$\therefore \rho_1 = 0.8\,\rho_2$

25 **(280)**

Let mass of the bullet be 'm' g. Total heat required to melt the bullet is,

$Q_1 = mc_l\Delta\theta + mL$

$= [m \times 0.03 \times (327 - 277)] + (m \times 5.5)$

$= (1.5 \times m) + (m \times 5.5)cal = (7m \times 4.20)J$

Loss in mechanical energy of the bullet due to obstacle

$= \frac{1}{2}(m \times 10^{-3})v^2 J$

Caution: As loss in energy is to be calculated in joule, mass of m gram needs to be converted into kg.

Now, 25% of this energy is absorbed by the obstacle.

∴Energy absorbed by the bullet,

$Q_2 = \frac{75}{100} \times \frac{1}{2} mv^2 \times 10^{-3} = \frac{3}{8} mv^2 \times 10^{-3} J$

The bullet will melt if $Q_2 \geq Q_1$

$\therefore \frac{3}{8} mv^2 \times 10^{-3} = 7m \times 4.20$

$\therefore v_{min}^2 = \frac{7m \times 4.20}{\frac{3}{8} m \times 10^{-3}} = 78400$

$\therefore v = 280 \text{ m/s}$

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LT 09 (JEE) Mathematics Solutions

: ANSWER KEY :

61)	b	62)	c	63)	b	64)	a	77)	a	78)	c	79)	a	80)	c
65)	c	66)	b	67)	a	68)	d	81)	12	82)	1	83)	6	84)	3
69)	a	70)	a	71)	c	72)	a	85)	0						
73)	c	74)	d	75)	c	76)	a								

: HINTS AND SOLUTIONS :

Single Correct Answer Type

61 **(b)**

$\int_a^b f(x)dx = [xf(x)]_a^b - \int_a^b xf'(x)dx (1)$

Now, put $f(x) = t \therefore x = f^{-1}(t)$

and $f'(x)dx = dt$and adjust the limits

Therefore, $\int_a^b f(x)dx = [bf(b) - af(a)] - \int_{f(a)}^{f(b)} f^{-1}(t)dt$ by (1)

$$\therefore \int_a^b f(x) + \int_{f(a)}^{f(b)} f^{-1}(x)dx = bf(b) - af(a)$$

62 **(c)**

$f(x) = \int_0^\pi \frac{t \sin t}{\sqrt{1+\tan^2 x \sin^2 t}} dt (1)$

Replacing t by $\pi - t$ and then adding $f(x)$ with equation (1)

$$f(x) = \frac{\pi}{2}\int_0^\pi \frac{\sin t}{\sqrt{1+\tan^2 x \sin^2 t}} dt$$

$$= \pi \int_0^{\pi/2} \frac{\sin t}{\sqrt{1+\tan^2 x(1-\cos^2 t)}} dt$$

$$= \pi \int_0^{\pi/2} \frac{\sin t}{\sqrt{\sec^2 x - \tan^2 x \cos^2 t}} dt$$

Let $y = \cos t$

$\therefore dy = -\sin t \, dt$

$$\Rightarrow f(x) = \pi \int_0^1 \frac{dy}{\sqrt{\sec^2 x - (\tan^2 x)y^2}}$$

$$= \frac{\pi}{\tan x}\int_0^1 \frac{dy}{\sqrt{\text{cosec}^2 x - y^2}}$$

$$= \frac{\pi}{\tan x}\left\{\sin^{-1}\frac{y}{\text{cosec}\, x}\right\}_0^1$$

$$= \frac{\pi}{\tan x}\sin^{-1}(\sin x) = \frac{\pi x}{\tan x}$$

63 **(b)**

$$I_1 = \int_{-100}^{101} \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$$

$$= \int_{-100}^{101} \frac{dx}{(5+2(1-x)-2(1-x)^2)(1+e^{2-4(1-x)})}$$

$$= 2I_1 = \int_{-100}^{101} \frac{dx}{5+2x-2x^2} = I_2$$

$$\Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$$

64 **(a)**

For $x \in \left(-\frac{\pi}{3}, 0\right)$, $2\cos x - 1 > 0$

$$\Rightarrow I = \int_{-\pi/3}^0 \frac{\pi}{2} dx = \frac{\pi^2}{6}$$

65 **(c)**

$$I = \int_0^{\sqrt{\text{In}\left(\frac{\pi}{2}\right)}} \cos(e^{x^2}) 2xe^{x^2} dx$$

Put $e^{x^2} = t \Rightarrow e^{x^2} 2x \, dx = dt$

$$\Rightarrow I = \int_1^{\pi/2} \cos t \, dt = [\sin t]_1^{\pi/2} = 1 - (\sin 1)$$

66 **(b)**

We have,

$$e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1} \, dt, x\epsilon(-1,1)$$

On differentiating w.r.t x, we get

$e^{-x}(f'(x) - f(x)) = \sqrt{x^4+1}$

$\Rightarrow f'(x) = f(x) + \sqrt{x^4+1}\, e^x$

$\because f^{-1}$ is the inverse of f

$\therefore f^{-1}(f(x)) = x$

$\Rightarrow f^{-1'}(f(x))f'(x) = 1$

$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f'(x)}$$

$$\Rightarrow f^{-1'}(f(x)) = \frac{1}{f(x) + \sqrt{x^4+1}\, e^x}$$

As $x = 0$, $f(x) = 2$

and $f^{-1}(2) = \frac{1}{2+1} = \frac{1}{3}$

67 **(a)**

$$\sum_{r=1}^n \int_0^1 f(r-1+x)dx$$

$$= \int_0^1 f(x)dx + \int_0^1 f(1+x)dx + \int_0^1 f(2+x)dx + \cdots + \int_0^1 f(n-1+x)dx$$

$$= \int_0^1 f(x)dx + \int_1^2 f(x)dx$$

$$+ \int_2^3 f(x)dx + \int_{r-1}^2 f(x)dx + \cdots$$

$$+ \int_{n-1}^1 f(x)dx = \int_0^n f(x)dx$$

68 **(d)**

$$I = \int_0^1 \frac{\tan^{-1} x}{x}\, dx$$

Putting $x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

$$\Rightarrow I = \int_0^{\pi/4} \frac{\theta}{\tan\theta} \sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4} \frac{2\theta}{\sin 2\theta} d\theta$$

Putting $2\theta = t$, i.e., $2d\theta = dt$,

We get $I = \frac{1}{2}\int_0^{\pi/2} \frac{t}{\sin t}\, dt$

$$= \frac{1}{2}\int_0^{\pi/2} \frac{x}{\sin x} dx$$

69 **(a)**

$I = \int_0^{\pi/2} \frac{\sin 2x}{x+1} dx$. Put $x = y/2$

$$\Rightarrow I = \int_0^{\pi} \frac{\sin y}{y+2} dy$$

$= \left(\frac{-\cos y}{y+2}\right)_0^{\pi} - \int_0^{\pi} \frac{\cos y}{(y+2)^2} dy$ (integrating by parts)

$$\Rightarrow I = \frac{1}{\pi+2} + \frac{1}{2} - A$$

70 **(a)**

Given, $f(x) = \int_1^x \sqrt{2-t^2}\, dt \Rightarrow f'(x) = \sqrt{2-x^2}\, dt \Rightarrow f'(x) = \sqrt{2-x^2}$

Also, $x^2 - f'(x) = 0$

$\therefore \quad x^2 = \sqrt{2-x^2} \Rightarrow x^4 = 2 - x^2$

$\Rightarrow \quad x^4 + x^2 - 2 = 0 \Rightarrow x = \pm 1$

71 **(c)**

Given $\quad I_1 = \int_{1-k}^{k} xf[x(1-x)]dx$

$\Rightarrow \quad I_1 = \int_{1-k}^{k} (1-x)f[(1-x)x]dx$

$= \int_{1-k}^{k} f[(1-x)]dx] - \int_{1-k}^{k} xf(1-x)]dx$

$\Rightarrow \quad I_1 = I_1 \quad \Rightarrow \frac{I_1}{I_2} = \frac{1}{2}$

72 **(a)**

$$\int_{-20\pi}^{20\pi} |\sin x|[\sin x]dx$$

$$= \int_0^{20\pi} |\sin x|([\sin x] + [-\sin x])\, dx$$

$$= -20\int_0^{\pi} (\sin x)dx = -20\,(-\cos x)_0^{\pi} = 20\,(-2)$$

$$= -40$$

73 **(c)**

$I = \int_0^{\pi} e^{\cos^2 x} \cos^3(2n+1)x\, dx, n \in Z \quad (1)$

$$= \int_0^{\pi} e^{\cos^2(\pi-x)} \cos^3[(2n+1)(\pi-x)]\, dx$$

$$\left[\text{Using} \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$$= \int_0^{\pi} e^{\cos^2 x} \cos^3[(2n+1)\pi - (2n+1)x]\, dx$$

$$= -\int_0^{\pi} -e^{\cos^2 x} \cos^3(2n+1)x\, dx$$

$= -I$

$\Rightarrow I = 0$

74 **(d)**

$$I = \int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}}$$

$$= \int \frac{dx}{\sqrt{\frac{\sin^3 x}{\cos^3 x}\cos^8 x}}$$

$$= \int \frac{\sec^4 x}{\sqrt{\tan^3 x}} dx$$

$$= \int \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan^3 x}} dx$$

Let $t = \tan x \Rightarrow dt = \sec^2 x\, dx$

$$\Rightarrow I = \int \frac{1+t^2}{t^{3/2}} dt$$

$$= \int \left(t^{-3/2} + t^{1/2}\right) dt$$

$$= -2t^{-\frac{1}{2}} + \frac{2}{3}t^{3/2} + C$$

$$= -2\sqrt{\cot x} + \frac{2}{3}\sqrt{\tan^3 x} + C$$

$$\Rightarrow a = -2, b = \frac{2}{3}$$

75 **(c)**

Let $x = t^6 \Rightarrow dx = 6t^5 dt$

$$\Rightarrow I = \int t^3\,(1+t^2)^4\, 6t^5 dt$$

$$\Rightarrow I = 6\int t^8(1+4t^2+6t^4+4t^6+t^8)\,dt$$

$$= 6\int (t^8+4t^{10}+6t^{12}+4t^{14}+t^{16})\,dt$$

$$= 6\left\{\frac{t^9}{9}+\frac{4t^{11}}{11}+\frac{6t^{13}}{13}+\frac{4t^{15}}{15}+\frac{t^{17}}{17}\right\}+C$$

$$= 6\left\{x^{2/3}+\frac{4}{11}x^{11/6}+\frac{6}{13}x^{13/6}+\frac{4}{15}x^{5/2}+\frac{1}{17}x^{17/6}\right\}+C$$

76 **(a)**

Here, $\int_0^{t^2}\{x\ f(x)\}dx = \frac{2}{5}t^5$

(Using Newton Leibnitz formula): differentiating both sides, we get

$$t^2\{f(t^2)\}.\left\{\frac{d}{dt}(t^2)\right\} - 0.f(0)\left\{\frac{d}{dt}(0)\right\} = 2t^4$$

$\Rightarrow\ t^2 f(t^2).2t = 2t^4$

$\Rightarrow\ f(t^2) = t$

$\therefore\ f\left(\frac{4}{25}\right) = \pm\frac{2}{5}$ $\quad\left[\text{putting } t = \pm\frac{2}{5}\right]$

$\Rightarrow\ f\left(\frac{4}{25}\right) = \frac{2}{5}$ $\quad$ [neglecting negative]

77 **(a)**

$$f'(x) = \frac{f(x)}{6f(x)-x}$$

Now $I = \int \frac{2x(x-6f(x))+f(x)}{(6f(x)-x)(x^2-f(x))^2}dx$

$$\Rightarrow I = -\int \frac{2x-f'(x)}{(x^2-f(x))^2}dx = \frac{1}{x^2-f(x)}+C$$

78 **(c)**

$$\int_{-1}^{1/2}\frac{e^x(2-x^2)dx}{(1-x)\sqrt{1-x^2}}$$

$$= \int_{-1}^{1/2}\frac{e^x(1-x^2+1)}{(1-x)\sqrt{1-x^2}}$$

$$= \int_{-1}^{1/2} e^x\left[\sqrt{\frac{1+x}{1-x}}+\frac{1}{(1-x)\sqrt{1-x^2}}\right]dx$$

$$= e^x\sqrt{\frac{1+x}{1-x}}\Bigg|_{-1}^{1/2}$$

$$= \sqrt{3e}$$

79 **(a)**

Let $I = \int_{-\pi/2}^{\pi/2}\frac{x^2\cos x}{1+e^x}\,dx$ ...(i)

$\left[\because \int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right]$

$\Rightarrow I = \int_{-\pi/2}^{\pi/2}\frac{x^2\cos(-x)}{1+e^{-x}}\,dx$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} x^2\cos x\left[\frac{1}{1+e^x}+\frac{1}{1+e^{-x}}\right]dx$$

$$= \int_{-\pi/2}^{\pi/2} x^2\cos x.(1)dx$$

$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx, when\ f(-x) = f(x)\right]$

$$\Rightarrow 2I = 2\int_0^{\pi/2} x^2\cos x\ dx$$

Using integration by parts, we get

$$2I = [x^2(\sin x)-(2x)(-\cos x)+(2)(-\sin x)]_0^{\pi/2}$$

$$\Rightarrow 2I = 2\left[\frac{\pi^2}{4}-2\right]$$

$$\therefore I = \frac{\pi^2}{4}-2$$

80 **(c)**

$$I_{4,3} = \int \cos^4 x\sin 3x dx$$

Integrating by parts, we have

$$I_{4,3} = -\frac{\cos 3x\cos^4 x}{3}-\frac{4}{3}\int \cos^3 x\sin x\cos 3x dx$$

But $\sin x\cos 3x = -\sin 2x+\sin 3x\cos x$, so

$$I_{4,3} = -\frac{\cos x\cos^4 x}{3}+\frac{4}{3}\int\cos^3 x\sin 2x dx-\frac{4}{3}\int\cos^4 x\sin 3x dx + C$$

$$= -\frac{\cos 3x\cos^4 x}{3}+\frac{4}{3}I_{3,2}-\frac{4}{3}I_{4,3}+C$$

Therefore, $\frac{7}{3}I_{4,3}-\frac{4}{3}I_{3,2} = -\frac{\cos 3x\cos^3 x}{3}+C$

Or $7I_{4,3}-4I_{3,2} = -\cos 3x\cos^4 x + C$

Integer Answer Type

81 **(12)**

$$f(f(x)) = f\left(\frac{x}{(1+x^n)^{\frac{1}{n}}}\right)\frac{\frac{x}{(1+x^n)^{\frac{1}{n}}}}{\left(1+\frac{x^n}{1+x^n}\right)^{\frac{1}{n}}}$$

$$= \frac{x}{(1+2x^n)^{\frac{1}{n}}} f(f(f(x)))$$

$$= f\left(\frac{x}{(1+2^n)^{\frac{1}{n}}}\right)$$

$$= \frac{\frac{x}{(1+2x^n)^{\frac{1}{n}}}}{\left(1+\frac{x^n}{1+2x^n}\right)^{\frac{1}{n}}} = \frac{x}{(1+3x^n)^{\frac{1}{n}}}$$

And so on.

$\Rightarrow (f\circ f\circ \ldots \circ f)(x) = \dfrac{x}{(1+nx^n)^{\frac{1}{n}}} = g(x)$

$\int g(x)x^{n-2}dx = \int \dfrac{x^{n-1}}{(1+nx^n)^{\frac{1}{n}}}dx$

$\Rightarrow I(10,x) = \int \dfrac{x^9}{(1+10x^{10})^{\frac{1}{10}}}dx$

Let $1+10x^{10} = t$

$\Rightarrow 100x^9dx = dtI(10,x)$

$= \dfrac{1}{100}\int \dfrac{1}{\left(t^{\frac{1}{10}}\right)}dt$

$= \dfrac{1}{100}\left(\dfrac{t^{1-\frac{1}{10}}}{1-\frac{1}{10}}\right)$

$= \dfrac{1}{90}t^{\frac{9}{10}} = \dfrac{1}{90}(1+10x^{10})^{\frac{9}{10}}$

$\Rightarrow I(10,1) = \dfrac{(11)^{10}}{90}$

$\Rightarrow p = 11, m = 9, k = 10, q = 90$

$\Rightarrow 10k - m - q + p$

$= 100 - 9 - 90 + 11 = 12$

82 **(1)**

$I^{r+1}(x) = l\underbrace{og\log\ldots\log(\log x)}_{r\text{ times}} = t$, say.

then

$dt = \dfrac{1}{xl(x)l^2(x)\ldots l'(x)}dx$

$\Rightarrow I(r,x) = \int dt = t + c = l^{r+1}(x) + c$

$\Rightarrow 1(3,e^{\ell}) = \left(\log\left(\log\left(\log(e^{\ell})\right)\right)\right)$

$= \log\left(\log(\log(e^e))\right)$

$= \log(\log(e'))$

$= \log e$

$=1$

83 **(6)**

Given $f^3(x) = \int_0^x t\cdot f^2(t)dt$

Differentiating, $3f^2(x)f'(x) = xf^2(x)$

$f(x) \neq 0 \;\therefore\; f'(x) = \dfrac{x}{3};\;\therefore\; f(x) = \dfrac{x^2}{6} + C$

But $f(0) = 0 \Rightarrow C = 0$

$f(6) = 6$

84 **(3)**

$\dfrac{d}{dx}(A\ln|\cos x + \sin x - 2| + Bx + C)$

$= A\,\dfrac{\cos x - \sin x}{\cos x + \sin x - 2} + B$

$= \dfrac{A\cos x - A\sin x + B\cos x + B\sin x - 2B}{\cos x + \sin x - 2}$

$\therefore 2 = A + B, -1 = -A + B, \lambda = -2B$

$\therefore A = 3/2, B = 1/2, \lambda = -1$

$\Rightarrow A + B + |\lambda| = 3$

85 **(0)**

$I = \int \dfrac{1}{\sqrt{2} + \cos x - \sin x}dx$

$= \dfrac{1}{\sqrt{2}}\int \dfrac{1}{1 + \left(\frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x\right)}dx$

$= \dfrac{1}{\sqrt{2}}\int \dfrac{1}{1 + \cos\left(\frac{\pi}{4} + x\right)}dx$

$= \dfrac{1}{2\sqrt{2}}\int \dfrac{1}{\cos^2\left(\frac{\pi}{8} + \frac{x}{2}\right)}dx$

$= \dfrac{1}{2\sqrt{2}}\int \sec^2\left(\dfrac{\pi}{8} + \dfrac{x}{2}\right)dx$

$= \dfrac{1}{2\sqrt{2}}\left(2\tan\left(\dfrac{\pi}{8} + \dfrac{x}{2}\right)\right) + c$

$= \dfrac{1}{\sqrt{2}}\tan\left(\dfrac{\pi}{8} + \dfrac{x}{2}\right) + c$

$\Rightarrow m = \dfrac{1}{\sqrt{2}}, n = \dfrac{1}{8}, a = \dfrac{1}{2}$

$\Rightarrow -a^2m^2 = \dfrac{1}{8} - \left(\dfrac{1}{2\sqrt{2}}\right) = \dfrac{1}{8} - \dfrac{1}{8} = 0$